Homework 1

- You are encouraged to discuss the homework problems with your classmates, but you **MUST** write you own solutions.
- Please type your homework in LaTex and submit it in class on Sep 27th.

Problem 1

Compute the center of the following groups

(1) GL_n

(2) D_{2n}

(3) S_n

(4) A_n

Problem 2

Let G be a group and H a subgroup of G which has index 2. Prove that H is a normal subgroup of G.

Problem 3

Let $\mathbb{F}_p$ denote the finite field of p elements. Compute the order of $\mathrm{GL}_n(\mathbb{F}_p)$ and $\mathrm{SL}_n(\mathbb{F}_p)$.

Problem 4

Let A and B be two groups and $C \trianglelefteq A$, and $D \trianglelefteq B$. Prove that there is an isomorphism of groups

$$(A \times B)/(C \times D) \simeq (A/C) \times (B/D).$$

Problem 5

Prove that if m and n are relatively prime integers, then $\mathbb{Z}/mn\mathbb{Z} \simeq \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$.

Problem 6

Let G be a group and H a subgroup of G which has index m. Let $N \leq H$ be the largest normal subgroup of G contained in H. Prove that there is a monomorphism $G/N \hookrightarrow \mathrm{S}_m$.